Instrument Parameters Affecting the Liquid/Gas Interface in XENON100

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Abstract: This note summarizes instrument parameters that affect the liquid/gas interface in XENON100. The consideration of dynamic pressure rather than static pressure explains why the level cannot be controlled by the linear feed-through as originally foreseen. It is found that the dynamic pressure increases quadratically with the flow. Its effect on the liquid level at typical recirculation speeds is too great to be corrected by the feed-through. A design modification, re-orientating the gas flow exiting the tube connected to the feed-through from the vertical into a horizontal direction, should remove this dynamical effect. Other important effects are related to the heat influx from the top, which keep the liquid level inside the bell below the outside level even in the absence of gas flow.

1 Dynamic Equilibrium inside the Diving Bell

Figure 1 shows a sketch of the situation affecting the liquid level in the detector, where we define the following terms:

p_{bell} : pressure inside the bell
p_{det} : detector operating pressure (outside the bell)
p_{stat} : static pressure
p_{dyn} : dynamic pressure (bulk motion of gas exiting the tube)
At $T_l = -95.0^\circ$ C, $P(T_l) = 2.026$ bar:
ρ_l : density of xenon liquid: $\rho_l = 2853\,\text{kg/m}^3$
ρ_v : density of xenon vapor: $\rho_v = 18.92\,\text{kg/m}^3$
$h_{\text{l,det}}$: liquid level outside the bell for $p_{\text{dyn}} > 0$
$h'_{\text{l,det}}$: liquid level outside the bell for $p_{\text{dyn}} = 0$

The following relation describes the dynamical equilibrium:

$$p_{\text{bell}} - p_{\text{det}} = p_{\text{stat}} + p_{\text{dyn}} = \rho_l\, g\, (h_{\text{l,det}} - h_{\text{l,bell}}) \quad (1)$$

$$p_{\text{stat}} = \rho_l\, g\, (h'_{\text{l,det}} - h_{\text{tube}}) \quad (2)$$

$$\approx 4.2 \times 10^3 \text{ Pa} = 42 \text{ mbar for } \Delta h_{\text{l}} \approx 15 \text{ cm}$$

$$p_{\text{dyn}} = \frac{1}{2}\rho_v v^2 = \frac{1}{2}\frac{j^2}{\rho_v} = \frac{1}{2}\frac{\dot{m}^2}{A^2 \rho_v} \quad (3)$$

$$\approx 4.1 \text{ Pa} \times \left(\frac{\Phi}{1 \text{ slpm}}\right)^2$$

$$p_{\text{dyn,max}} \approx 1.5 \times 10^2 \text{ Pa } (\Phi = 6 \text{ slpm}) \approx 3.5\%\, p_{\text{stat}}$$

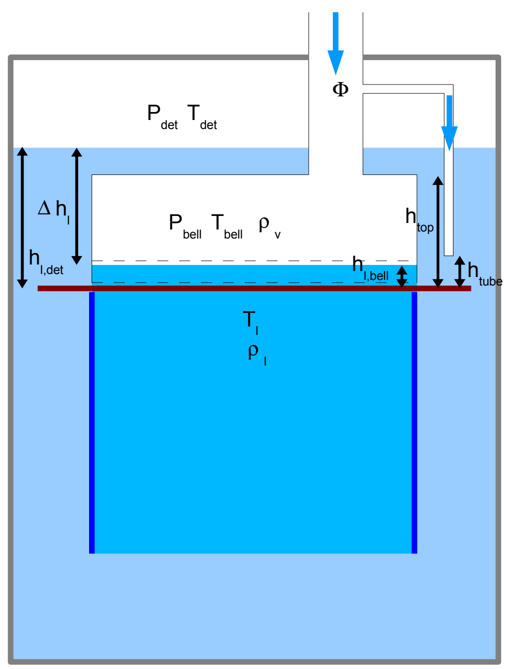

Fig. 1: Schematic of liquid levels.

Even though the dynamic pressure is small compared to the static pressure, it has an important effect on the liquid level at full recirculation speed: $\delta h_{\mathrm{max}} \approx 5$ mm

Notice that equation 1 neglected the removal of excess gas pressure by phase transition at the liquid/gas interface. This should be a good assumption at the relevant gas flows, since the heat exchange between the top layers of the liquid inside the bell and the liquid xenon outside the bell should be small. Effectively, this means that the recirculation will keep the top liquid layers inside the bell at a slightly higher temperature than the liquid outside the bell. This assumption is also justified by the observed thermal effects described in section 1.3.

1.1 Dependency between the Liquid Levels inside and outside the Bell

The liquid level outside (usually on top of) the bell, $h_{\mathrm{l,det}}$, is given by the continuity equation of mass conservation. We consider only the mass $M_{\mathrm{Xe}}^{\mathrm{up}}$ above the upper copper plate, which closes the bell, with the implicit assumption that little is changed in the main TPC volume and in the side and bottom veto volumes by changes in the liquid level inside the bell:

$$M_{\mathrm{Xe}}^{\mathrm{up}} = \rho_{\mathrm{l,bell}} A_{\mathrm{bell}} h_{\mathrm{l,bell}} + \rho_{\mathrm{l,det}} (A_{\mathrm{det}} - A_{\mathrm{bell}}) h_{\mathrm{top}} + \rho_{\mathrm{l,det}} (A_{\mathrm{det}} - A_{\mathrm{conduit}})(h_{\mathrm{l,det}} - h_{\mathrm{top}}) \tag{4}$$

For completeness, we have here distinguished between the liquid density inside the bell and outside, which may be slightly different due to the temperature dependence of the density. However, we will neglect these higher order effects in the following, and return to the assumption $\rho_{\mathrm{l,bell}} = \rho_{\mathrm{l,det}} \equiv \rho_l$. Hence follows:

$$\begin{aligned} h_{\mathrm{l,det}} &= h_{\mathrm{l,det}}^{\mathrm{max}} - a\, h_{\mathrm{l,bell}} \\ \text{with} \quad h_{\mathrm{l,det}}^{\mathrm{max}} &\stackrel{\mathrm{def}}{=} \frac{M_{\mathrm{Xe}}}{\rho_l (A_{\mathrm{det}} - A_{\mathrm{conduit}})} + \frac{A_{\mathrm{bell}} - A_{\mathrm{conduit}}}{A_{\mathrm{det}} - A_{\mathrm{conduit}}} h_{\mathrm{top}} \\ a &\stackrel{\mathrm{def}}{=} \frac{A_{\mathrm{bell}}}{A_{\mathrm{det}} - A_{\mathrm{conduit}}} \end{aligned} \tag{5}$$

Therefore, the liquid level inside the bell depends linearly on the level outside the bell - and vice versa. The sign of the dependency is negative, hence a higher liquid level outside means a lower liquid level inside, with a factor a as proportionality constant.

1.2 Dependency of the Liquid Level inside the Diving Bell on the Gas Flow

Equation 3 made use of the mass flux through the tube:

$$j = \rho_v v = \frac{\dot{m}}{A} \tag{6}$$

As an ancillary result, one computes the velocity of xenon vapor in the SS tube if its inner diameter is known. Assuming $d = 0.125$ in $\Leftrightarrow A_{\mathrm{tube}} = 7.92 \times 10^{-6}$ m^2:

$$v = 0.66\, \frac{\mathrm{m}}{\mathrm{s}} \times \Phi \text{ [slpm]}$$

The mass flow $\dot{m}$ and its equivalent as measured by the flowmeter in volumetric units Φ are converted via:

$$\begin{aligned}\Phi\,[\text{std L/min}] &= \frac{\dot{m}\,[\text{kg/s}]}{\rho_{\text{std,Xe}}\,[\text{kg/m}^3]} \times 10^3\,[\text{L/m}^3] \times 60\,[\text{s/min}] \;=\; 10173\,\dot{m}\,[\text{kg/s}] \\ \rho_{\text{std,Xe}} &= 5.898\,\text{kg/m}^3 \quad \text{at } T = 0^\circ\,\text{C and } p = 101325\text{ Pa}\end{aligned} \tag{7}$$

From equations 1 and 2, one can write:

$$p_{\text{dyn}} = \rho_l\, g\, (\underbrace{h_{\text{tube}} - h_{\text{l,bell}}}_{\delta h_{\text{l,bell}}} + \underbrace{h_{\text{l,det}} - h'_{\text{l,det}}}_{\delta h_{\text{l,det}}}) \tag{8}$$

$$\text{from Eqn 5:} \quad \delta h_{\text{l,det}} = a\, \delta h_{\text{l,bell}} \qquad \text{(for liquid level outside above the bell)}$$

$$\Rightarrow \quad p_{\text{dyn}} = \rho_l\, g\, (1+a)\, \delta h_{\text{l,bell}} = \frac{1}{2} \frac{\dot{m}^2}{A^2_{\text{tube}}\, \rho_v} \tag{9}$$

using equation 3. With equation 7 follows:

$$\begin{aligned} h_{\text{l,bell}} &= h_{\text{tube}} - \frac{\dot{m}^2}{2\rho_v \rho_l g (1+a) A^2_{\text{tube}}} = h_{\text{tube}} - \frac{\Phi^2}{2\rho_v \rho_l g (1+a) A^2_{\text{tube}} \times (10173)^2} \\ &\approx h_{\text{tube}} - \frac{0.145\text{ mm}}{1+a} \times \left(\frac{\Phi}{1\text{ slpm}}\right)^2 \qquad \text{for } p_{\text{det}} - p_{\text{bell}} \geq p_{\text{stat}} \end{aligned} \tag{10}$$

Equation 10 shows that, within the assumptions, the liquid level $h_{\text{l,bell}}$ depends on two parameters: the position of the tube opening h_{tube}, controlled by the linear feed-through, and quadratically on the flow Φ. It does not depend on the absolute liquid level on top of the bell.(However, the maximum recirculation flow is reduced with increasing liquid level due to the enhanced heat influx.) The quadratic flow dependency reduces the liquid level by $< 3\%$ of the mesh-anode distance at a flow of 1 slpm, but this reduction can be greater than 100% at a flow of 6 slpm, since $(1+a) < 2$, pushing the liquid level below the gate mesh. Notice that the geometric factors are only estimated here. Exact values can be extracted from early measurements before the long levelmeter got damaged.

A design modification re-orientating the gas flow exiting the tube connected to the feed-through from the vertical into a horizontal direction, should remove this dynamical effect. For instance, a plate attached to the tube that is connected to the linear feed-thru and facing the opening of the tube, could accomplish this. An alternative would be the use of a wider diameter tube, which would reduce the gas speed and reduce the dynamical effect to acceptable levels.

1.3 Thermal Effects

At zero gas flow $\Phi = 0$, the system could be expected to return to thermal equilibrium, i.e., the pressure difference $p_{\text{det}} - p_{\text{bell}}$ would become zero, and the liquid levels inside and outside the bell would equalize. It is observed that this is not the case. While the pressure difference may drop below p_{stat}, it reaches a minimum pressure difference Δp_{min}, maintaining a level difference great enough that the short levelmeters never get fully covered by liquid. This means that Δp_{min} is comparable in size to p_{stat}: $\Delta p_{\text{min}} \lesssim p_{\text{stat}}$. (It cannot be greater than p_{stat}.) This observation can be explained by heat flowing in through the cables of the top PMTs and through the large stainless steel pipe forming the cable conduit and the gas input connection.

The heat influx $\dot{Q}_w$ results in a small increase ΔT of the vapor temperature. This drives a heat loss $\dot{Q}_c$ through the SS wall of the bell and the copper plate, as well as through the liquid via condensation and convection, until an equilibrium temperature difference is reached. The pressure increase corresponding to ΔT depends on the respective time scales of these processes. The condensation time scale should be by far the fastest but the others are more difficult to estimate. For instance, if convection in the gas phase were fast compared to the time scales of heat conduction through the metals and convection in the liquid, a uniform gas temperature with $T_{\mathrm{det}} + \Delta T$ would be established, and a correspondingly warmer liquid layer would form inside the bell. This would place the pressure on the vapor curve, and result in the largest pressure increase for a given ΔT, e.g., a temperature increase by only 0.5° C corresponds to a pressure increase of ~ 50 mbar in this case. More likely, however, gas convection is not very effective, since there is no gas flow. As a result, a temperature gradient forms across the gas volume of the bell, and the pressure increase would be closer to an isochoric process for the average temperature increase in the bell. In this case, an average temperature increase of 1° C would result in a pressure increase of ~ 13 mbar.

Which role do thermal effects play with $\Phi > 0$? At large flow, the assumption of rapid heat transport in the gas is a good approximation, and the gas flow effectively removes the heat from the bell. If, however, the flow is decreased in order to minimize the effect of dynamic pressure on the liquid level, the heat influx will result in warming of the gas, and add to its pressure. The latter is removed by the gas flow through the tube.