Status of the Bayesian analysis of XENON100 data

- Reconstruction of Dark Matter parameters -

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Concepts of statistical inference

Frequentist (classical) approach	Bayesian approach
The performance of statistical procedures is determined in the long-run over an infinite number of hypothetical repetitions of an experiment	Statements about the probability of a parameter must be interpreted as → "*degree of belief*", the prior distribution can be subjective

The posterior probability density function is constructed using Bayes' theorem:

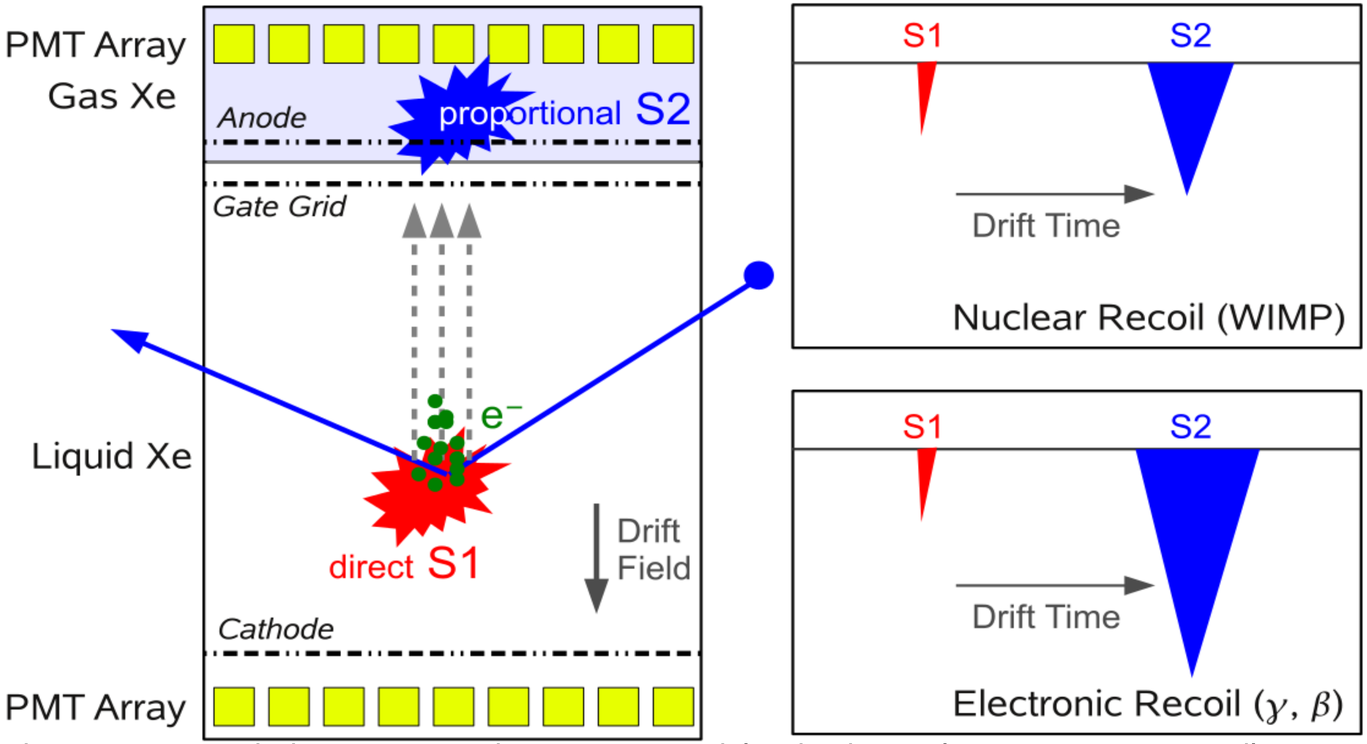
$$\mathcal{P}(\mathrm{H}|\mathrm{E}) \propto \pi(\mathrm{H})\mathcal{P}(\mathrm{E}|\mathrm{H})$$

posterior — $\mathcal{P}(\mathrm{H}|\mathrm{E})$; prior — $\pi(\mathrm{H})$; likelihood — $\mathcal{P}(\mathrm{E}|\mathrm{H})$

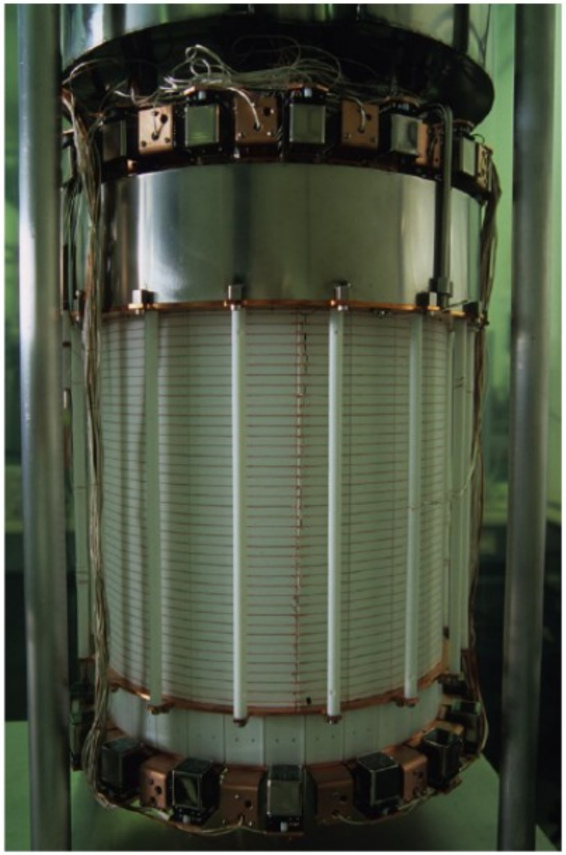
A ***Frequentist*** is a person whose long-run ambition is to be wrong 5% of the time.

A ***Bayesianist*** is a person who, vaguely expecting a horse, and catching a glimpse of a donkey, strongly believes he has seen a mule. (Senn, 1997)

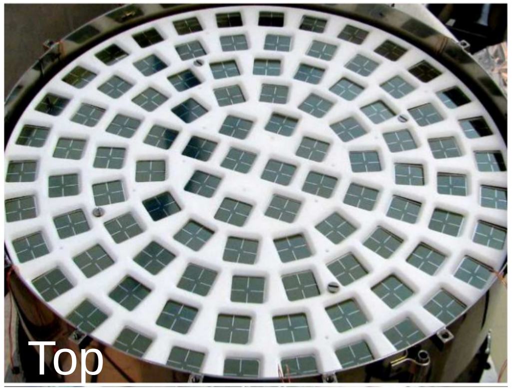
XENON100 detector

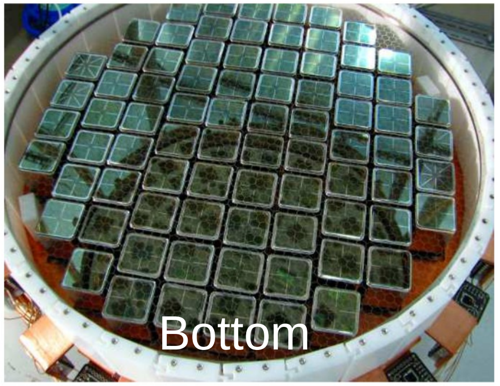


The XENON100 dark matter experiment, Astroparticle Physics, Volume 35, Issue 9, April 2012



- **S1**: primary scintillation light
- **S2**: proportional scintillation charge signal
- **S2/S1**: holds information about interaction
  → NR and ER discrimination
- **S1**, **S2**: information about deposited energy





Differences to standard analysis

What is new?

- Using S1 as well as S2 for energy reconstruction

What has been improved?

- Using spatially dependent likelihood rather than spatially averaged one

Statistics?

- Consistent treatment of all sources of error via incorporation in the likelihood in parameterized form and marginalization over nuisance parameters

Spatially dependent signal model

$E, \vec{r}$

$p_1(S1|E,\vec{r})$ → $\varepsilon_1(S1)$ → **S1**

$p_2(S2|E,\vec{r})$ → $\varepsilon_2(S2)$ → **S2**

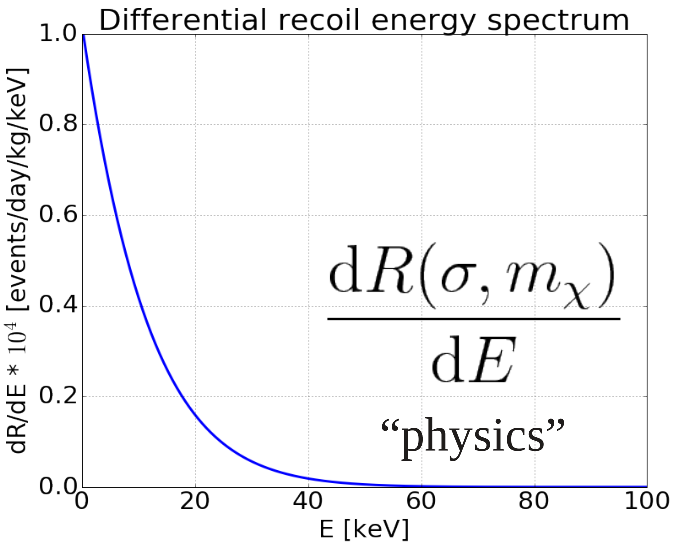
Energy deposition at location r

Conversion to charge & light + Detector response

Efficiencies (not implemented yet)

Measured quantities in PE

Measurement process



$$\frac{\mathrm{d}R(\sigma, m_\chi)}{\mathrm{d}E}$$

"physics"

$$p\,(S1, S2|E,\vec{r}) \rightarrow \frac{d^2R(S1,S2;m,\sigma,\vec{r})}{\mathrm{d}S1\mathrm{d}S2}$$

signal model

DM parameter reconstruction

I. Prepare MC mock dataset for testing

1. Calculate expected number of events **n** assuming WIMP interactions are uniformly distributed in the detector (scaled with an exposure and reference cross-section)

2. From the expected number of events **n** we do a Poissonian draw to get the number of observed events **N**

3. Draw **N** random energies following the "real" recoil energy spectrum

4. Generate **N** tuples of $\{\mathbf{S1, S2}, \vec{r}\}_{i=1,...,N}$ for the drawn recoil energies

II. DM parameter estimation

5. Feed $\{\mathbf{S1, S2}, \vec{r}\}_{i=1,...,N}$ into a MCMC sampler, using Bayes' theorem to calculate the posterior and reconstruct mass and cross section (uniform priors in $\log_{10}$) by integrating out the unknown Energy deposit

DM parameter reconstruction

To use Bayes' theorem to calculate a posterior, we first have to find a likelihood function:

$$\Theta = \{m_\chi, \sigma_\chi\} \quad , \quad S = \{S1, S2\}_{i=1,\dots,N}$$

DM Parameters:
mass and cross-section

Measured signals at position $\vec{r}_i$

$$\mathcal{L}(\Theta, \vec{r}) = p(S, N|\Theta, \vec{r})$$

Likelihood of observing N events with signal S, given the DM parameters and interaction position

$$p(S|N, \Theta, \vec{r})$$

$$\prod_i \int_0^\infty p(E|\Theta) \cdot p_1(S1_i|E, \vec{r}_i) \cdot p_2(S2_i|E, \vec{r}_i) \mathrm{d}E$$

$$p(N|\Theta)$$

$$\mathrm{Poi}_N(n)$$

$$n = \int_{S1_{\min}}^{S1_{\max}} \int_{S2_{\min}}^{S2_{\max}} \frac{\mathrm{d}^2 R(S1, S2; m_\chi, \sigma_\chi)}{\mathrm{d}S1\mathrm{d}S2} \mathrm{d}S1\mathrm{d}S2$$

PDFs for S1 & S2

$$cS1 = \frac{\langle \hat{\mu} \rangle}{\hat{\mu}(\vec{r})} \cdot S1$$

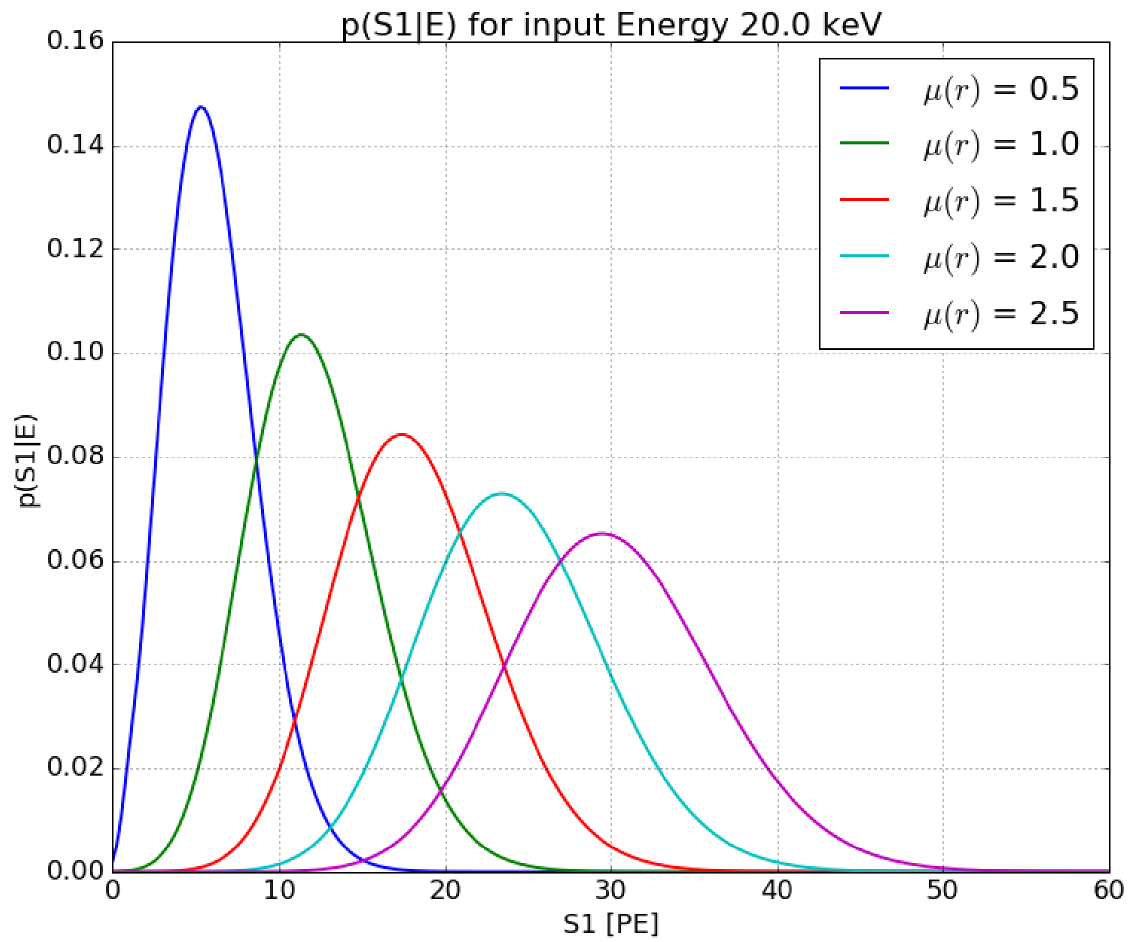
The standard approach uses spatially averaged values of the light yield and light collection efficiency.

The spatial dependence is present in the "relative light collection efficiency" μ(r):

$$\mathcal{L}_y(\vec{r}) = \frac{\hat{\mu}(\vec{r})}{\langle \hat{\mu} \rangle} \langle \mathcal{L}_y \rangle = \mu(\vec{r}) \langle \mathcal{L}_y \rangle$$

↓

$$p_1(S1|E, \vec{r}) = p_1(S1|E, \mu(\vec{r}))$$



PDFs for S1 & S2

$$cS2 = \frac{\langle \hat{\delta} \rangle}{\hat{\delta}(x,y)} \exp(\frac{t_d(z)}{\tau_e}) \cdot S2$$

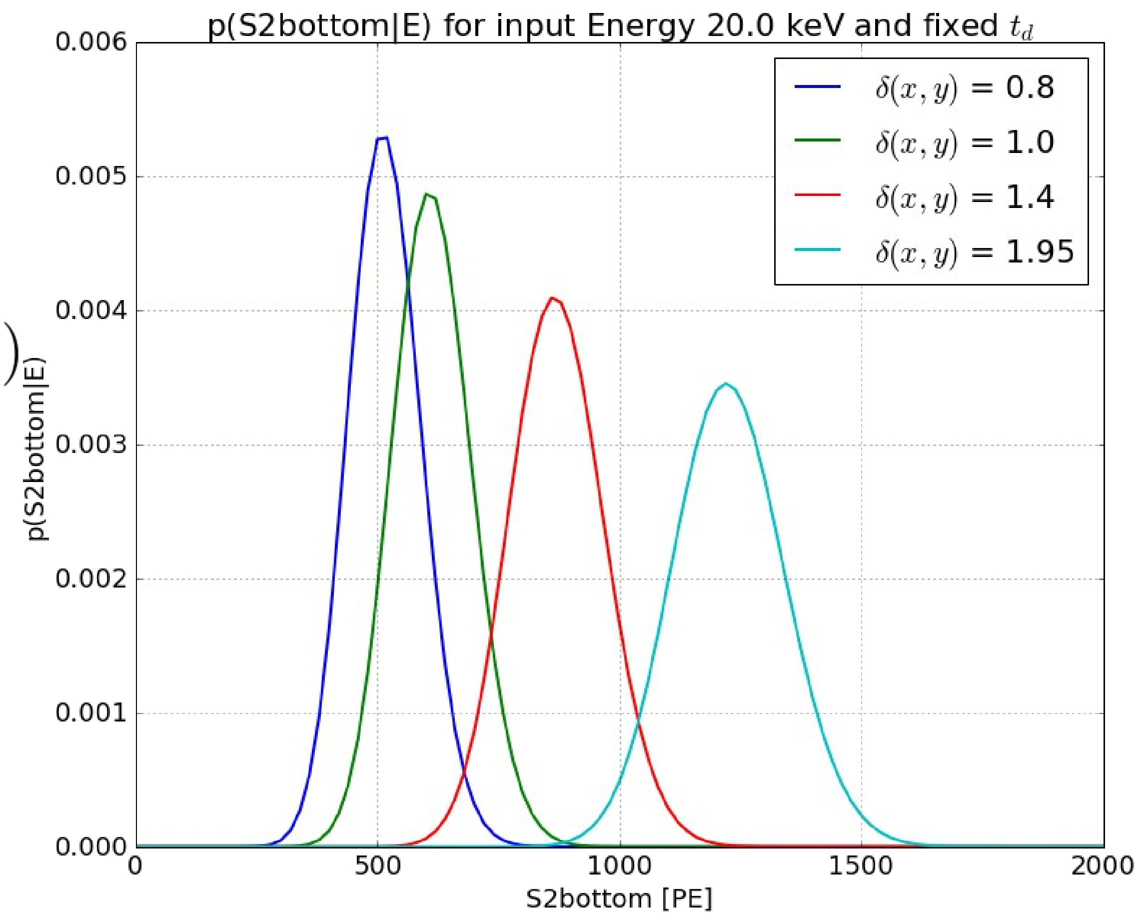
The standard approach already incorporates the corrections for extraction yield and drift time.

In the spatially dependent case, we get different pdfs for different values of extraction yield at location x,y and drift time (depth z).

$$p_2(S2|E,\vec{r}) = p_2(S2|E,\delta(x,y),t_d(z))$$

extraction efficiency
x
prop. scintillation gain

drift time
(τ_e = 500ms)



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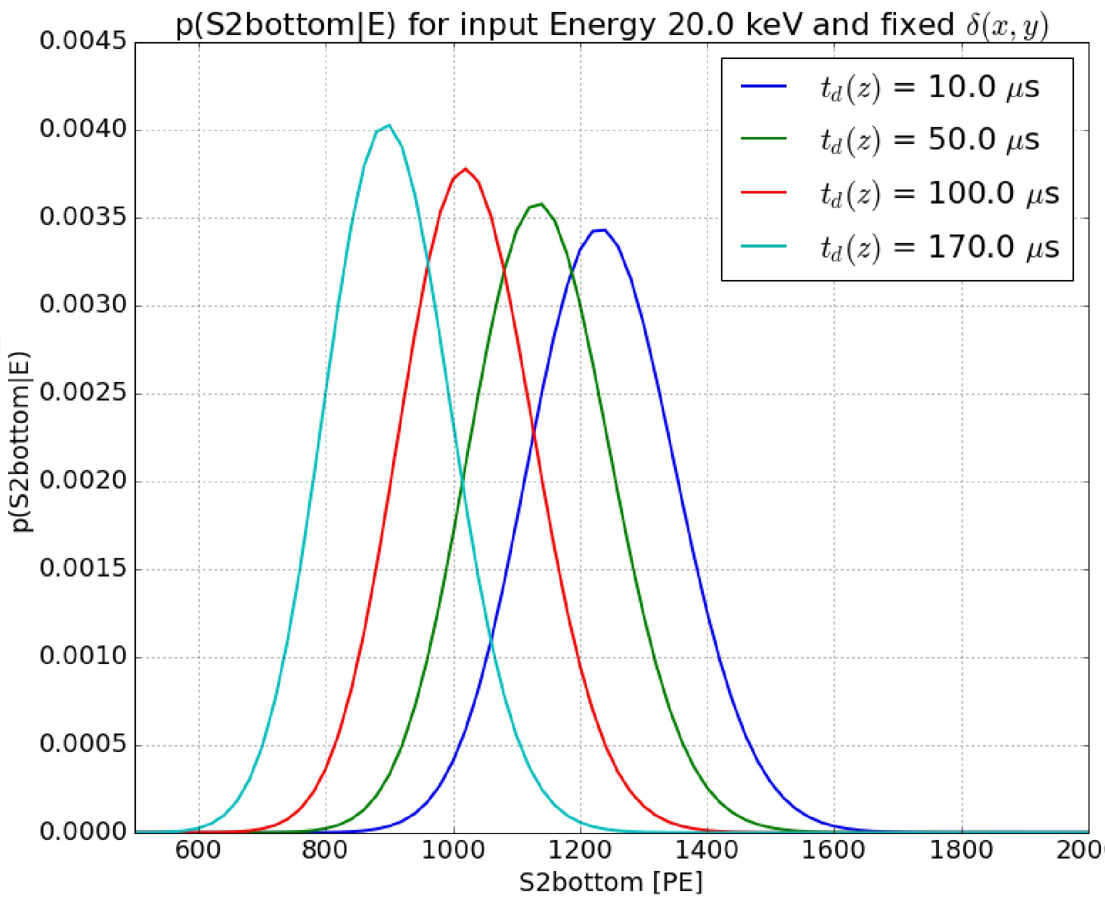
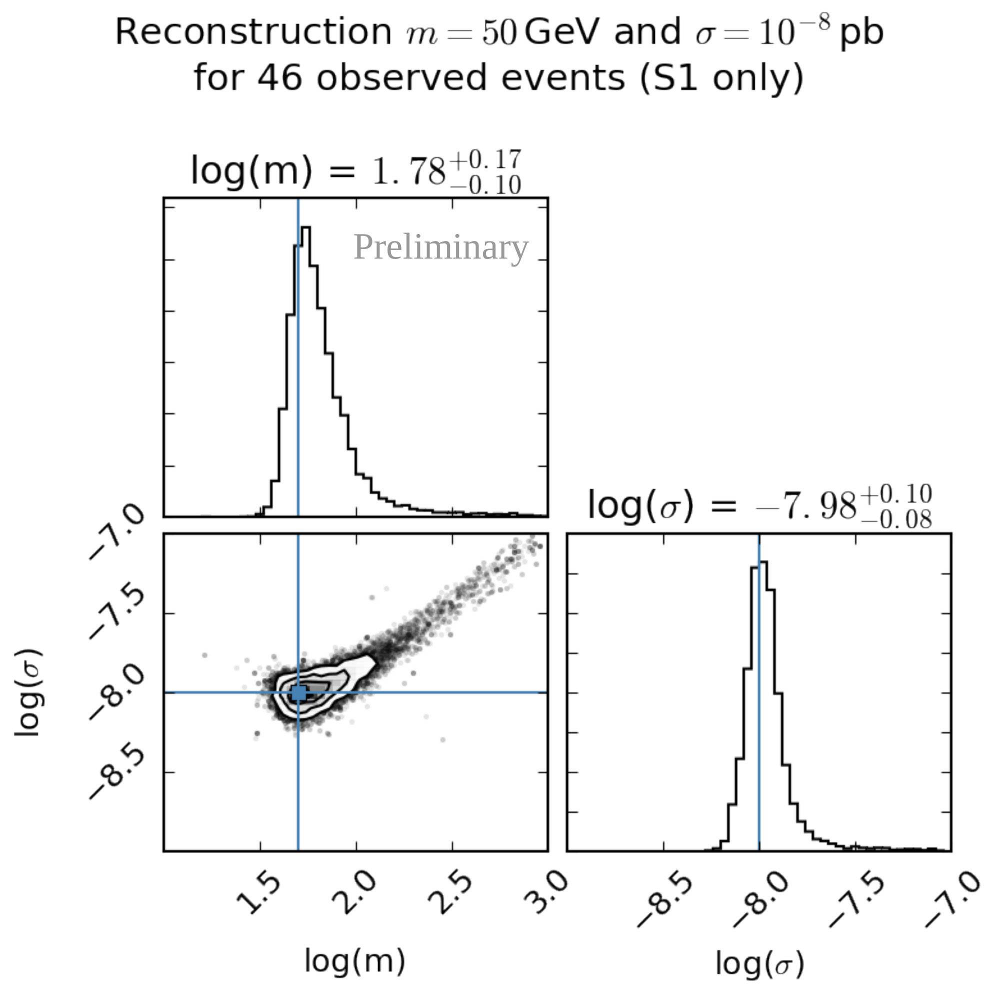


Results of DM parameter reconstruction

Results for a WIMP with mass 50 GeV and a cross-section of 10^{-8} pb, resulting in n=46,57 expected and N=47 observed events in the Xenon100 detector.
(Priors for mass and cross-section are chosen flat in $\log_{10}$-space)



Summary & Outlook

Summary

- Construction of a spatially dependent signal model
- Reconstructing Dark Matter parameters using a new spatially dependent signal model

Outlook

- Prepare and apply efficiencies of the Xenon100 detector
- Include a background model
- Use real Xenon100 (or even Xenon1T) data to make inferences on parameters